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Argumentative Causal Discovery

Extract Causal Relations from Data with Assumption-based Argumentation

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Outline

Introduction

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Motivation

- Measuring *Causal Effects* is necessary when one wants to evaluate the impact of actions e.g. the effect of a treatment on a patient's health

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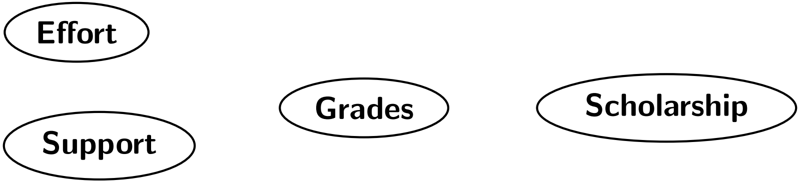
The need to extract as much causal information as possible from observational data

Student's Achievement Example



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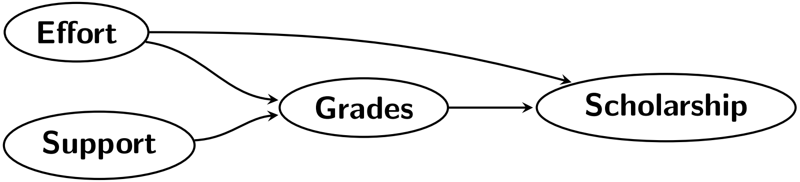

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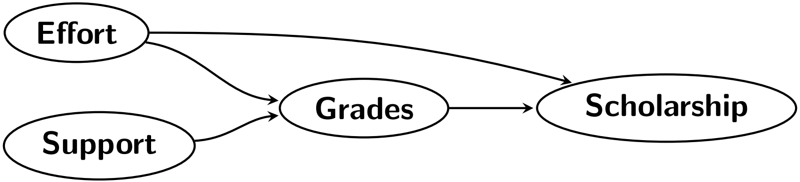

- Effort $\perp\!\!\!\perp$ Support
- Effort $\perp\!\!\!\perp$ Support | Grades
- Effort $\perp\!\!\!\perp$ Support | Scholar
- Effort $\perp\!\!\!\perp$ Support | {Grades, Scholar}
- Support $\not\!\perp\!\!\!\perp$ Scholar
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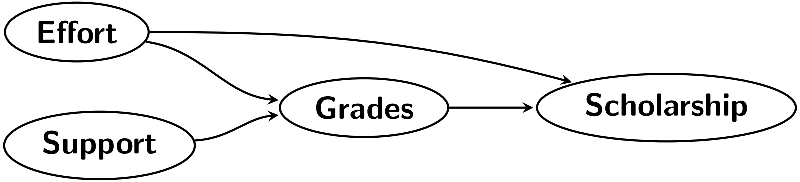

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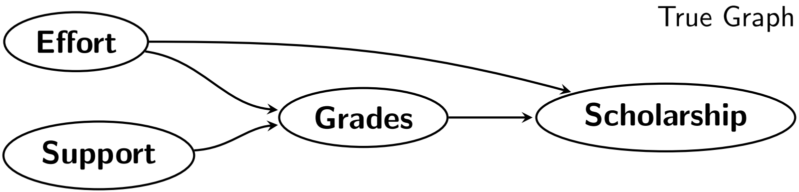
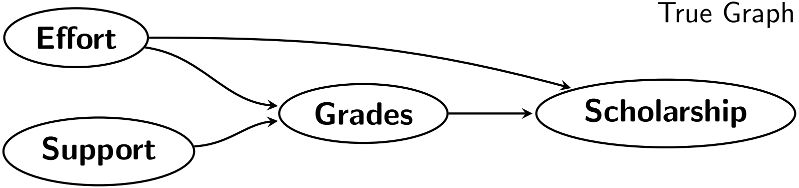
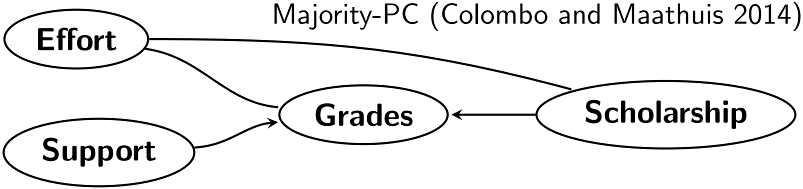


Student's Achievement Example

True Graph



Majority-PC (Colombo and Maathuis 2014)



Causal ABA

Assumption-based Argumentation (ABA)

An ABA framework (ABAF) is a tuple $(\mathcal{L}, \mathcal{R}, \mathcal{A}, {}^{-})$, where

- $(\mathcal{L}, \mathcal{R})$ is a *deductive system*,
- $\mathcal{A} \subseteq \mathcal{L}$ a set of *assumptions*, and
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- A set $S \subseteq \mathcal{A}$ *attacks* $T \subseteq \mathcal{A}$ if there is $S' \subseteq S$, $a \in T$, s.t. $S' \vdash \bar{a}$.
- S is **conflict-free** if it does not attack itself;
- S is a **Stable Extension** iff S is conflict-free and attacks each $\{x\} \subseteq \mathcal{A} \setminus S$.

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1. Enforce acyclicity
2. Enforce d-separation
3. Allow external input

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Let $D_{dag} = (\mathcal{A}_{dag}, \mathcal{R}_{dag}, ^{-})$ where our assumptions:

$$\mathcal{A}_{dag} = \{arr_{xy} \mid x, y \in \mathbf{V}, x \neq y\} \cup \{noe_{xy} \mid x, y \in \mathbf{V}, x \neq y\}$$

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and rules $\mathcal{R}_{dag}$:

- $\overline{a} \leftarrow b,\ a \neq b,\ a, b \in \{arr_{xy}, arr_{yx}, noe_{xy}\},\ x, y \in \mathbf{V} \implies$ **Can only choose one assumption**

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Stable Extensions represent DAGs, one per extension.

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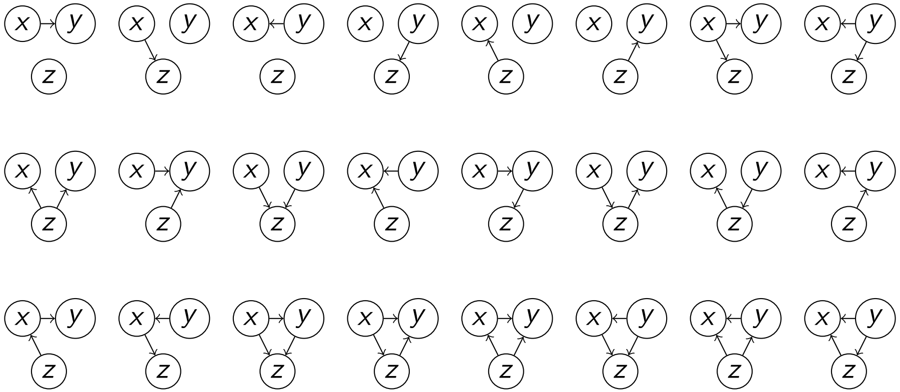
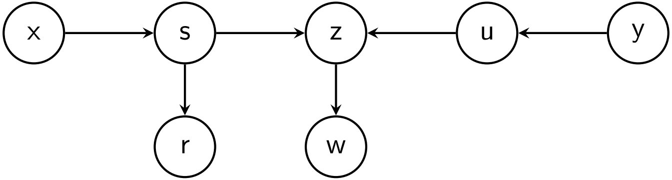
All Possible DAGs with Three Variables (x, y, z)



Connect DAGs and Data: d-Separation (Pearl 2009)



Unconditional

$x \not\perp\!\!\!\perp_{\mathcal{G}} z$

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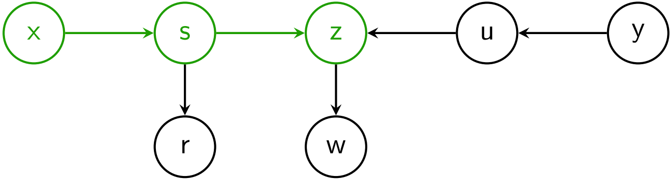

Unconditional

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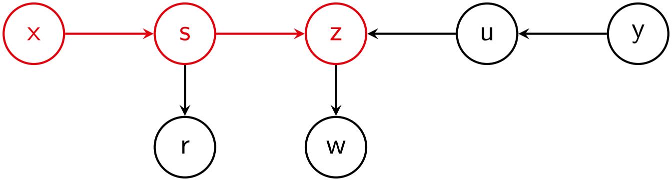

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d-separate by conditioning

$x \perp\!\!\!\perp_{\mathcal{G}} z \mid s$ Blocked

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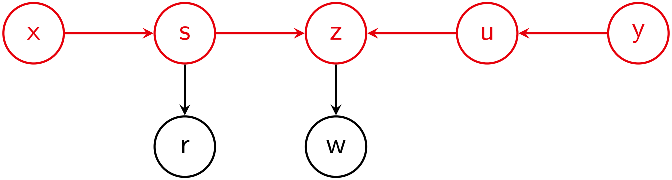

Unconditional

$x \not\perp\!\!\!\perp_{\mathcal{G}} z$

$x \perp\!\!\!\perp_{\mathcal{G}} y$ Blocked

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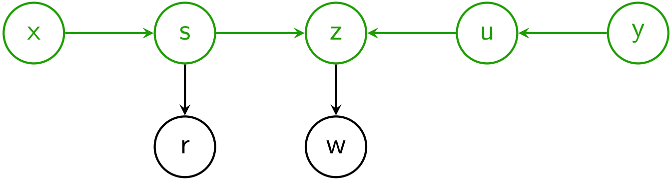

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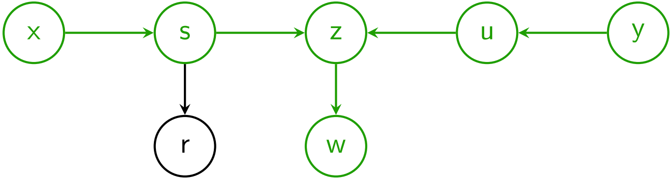

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d-connect by conditioning

$x \not\perp\!\!\!\perp_{\mathcal{G}} y \mid z$

$x \not\perp\!\!\!\perp_{\mathcal{G}} y \mid w$ Active

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2. d-Separation in ABA

We assume independence between variables:

$$\mathcal{A}_{ds} = \{(x \perp\!\!\!\perp y \mid \mathbf{Z}) \mid \mathbf{Z} \subseteq \mathbf{V}, x, y \in \mathbf{V} \setminus \mathbf{Z}, x \neq y\}.$$

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We add the following rules ($\mathcal{R}_{ds}$):

$$\begin{array}{rll} dpath_{xy} \leftarrow arr_{xy} & dpath_{xz} \leftarrow dpath_{xy}, arr_{yz} & \\ e_{xy} \leftarrow arr_{xy} & e_{xy} \leftarrow arr_{yx} & \overline{noe_{xy}} \leftarrow e_{xy} \\ & \overline{x \perp\!\!\!\perp y \,|\, \mathbf{Z}} \leftarrow p_{\mathbf{Z}} & \end{array}$$

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Each stable extension will contain a DAG and the independences that this entail via d-separation.

$$dpath_{xy} \leftarrow arr_{xy} \qquad dpath_{xz} \leftarrow dpath_{xy}, arr_{yz}$$

$$e_{xy} \leftarrow arr_{xy} \qquad e_{xy} \leftarrow arr_{yx} \qquad \overline{noe_{xy}} \leftarrow e_{xy}$$

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3. Input External Causal Knowledge

Suppose we observe

$$x \not\perp\!\!\!\perp y \mid \{z\}$$

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with contraries $\overline{bp_{p|\mathbf{Z}}} = ap_{p|\mathbf{Z}}$ and add the following rules ($\mathcal{R}_{ext}$):

- $x \perp\!\!\!\perp y \mid \mathbf{Z} \leftarrow bp_{p_1|\mathbf{Z}}, \ldots, bp_{p_k|\mathbf{Z}}$ where $p_1, \ldots, p_k$ denote all paths between x and y;
- $ap_{p|\mathbf{Z}} \leftarrow p$ for each $\mathbf{Z}$-active p

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Causal ABA becomes non-flat.

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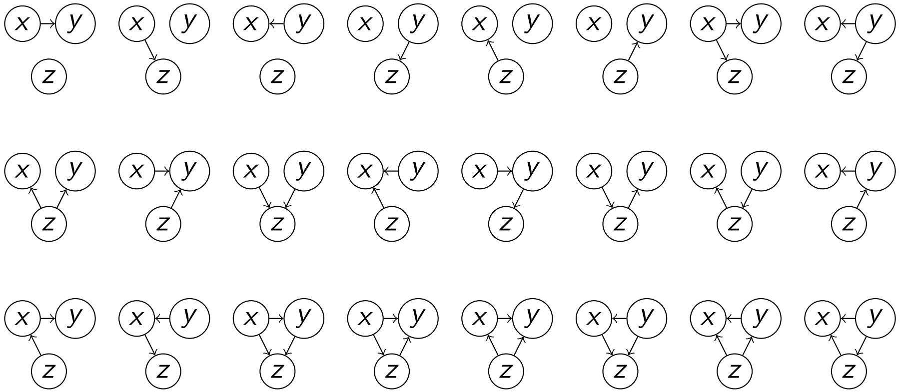

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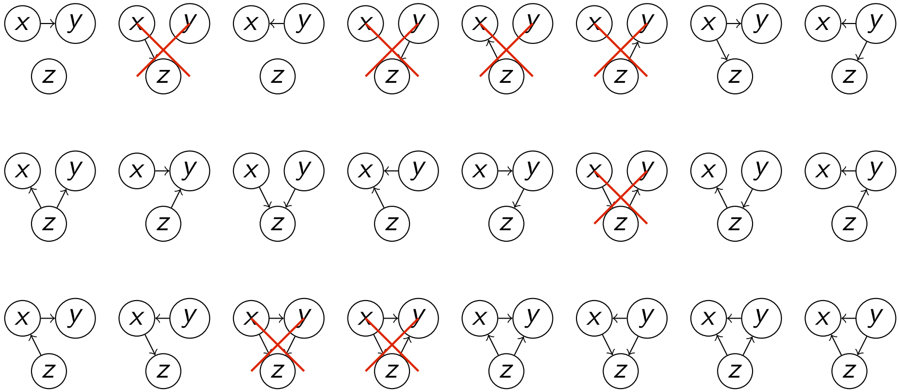
Three Variables DAGs compatible with $x \not\perp\!\!\!\perp y \mid \{z\}$



ABA-PC

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Instantiation & Implementation

- We propose an end-to-end causal discovery algorithm called **ABA-PC**
  - We experiment feeding the independence statements calculated by Majority-PC (Colombo and Maathuis 2014)
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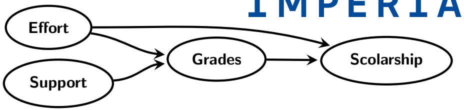
- We implement our Causal ABAF in clingo (Gebser et al. 2019)
- Leverage the correspondence between ABA and Answer Set Programming (Rapberger, Ulbricht and Toni 2024) to calculate stable sets

Student's Achievement Example



$\mathrm{Ef} \not\perp\!\!\!\perp \mathrm{Sc}$	$p = 0.00$	$\mathcal{S} = 1.00$			
$\mathrm{Ef} \not\perp\!\!\!\perp \mathrm{Gr}$	$p = 0.00$	$\mathcal{S} = 1.00$	$\mathrm{Gr} \not\perp\!\!\!\perp \mathrm{Sc} \mid \{Ef\}$	$p = 0.00$	$\mathcal{S} = 0.50$
$\mathrm{Su} \not\perp\!\!\!\perp \mathrm{Gr}$	$p = 0.00$	$\mathcal{S} = 1.00$	$\mathrm{Gr} \not\perp\!\!\!\perp \mathrm{Sc} \mid \{Su\}$	$p = 0.00$	$\mathcal{S} = 0.50$
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$\mathrm{Su} \perp\!\!\!\perp \mathrm{Ef}$	$p = 0.46$	$\mathcal{S} = 0.71$	$\mathrm{Su} \not\perp\!\!\!\perp \mathrm{Sc} \mid \{Gr\}$	$p = 0.03$	$\mathcal{S} = 0.35$
$\mathrm{Su} \not\perp\!\!\!\perp \mathrm{Sc}$	$p = 0.05$	$\mathcal{S} = 0.52$	$\mathrm{Su} \perp\!\!\!\perp \mathrm{Ef} \mid \{Gr\}$	$p = 0.33$	$\mathcal{S} = 0.32$
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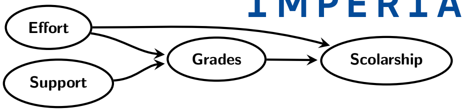

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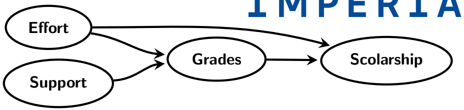

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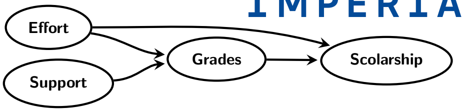
$\mathrm{Su} \not\perp\!\!\!\perp \mathrm{Sc}$	$p = 0.05$	$\mathcal{S} = 0.52$
$\mathrm{Ef} \not\perp\!\!\!\perp \mathrm{Sc} \mid \{Su\}$	$p = 0.00$	$\mathcal{S} = 0.50$
$\mathrm{Ef} \not\perp\!\!\!\perp \mathrm{Sc} \mid \{Gr\}$	$p = 0.00$	$\mathcal{S} = 0.50$
$\mathrm{Ef} \not\perp\!\!\!\perp \mathrm{Gr} \mid \{Su\}$	$p = 0.00$	$\mathcal{S} = 0.50$
$\mathrm{Su} \not\perp\!\!\!\perp \mathrm{Gr} \mid \{Sc\}$	$p = 0.00$	$\mathcal{S} = 0.50$
$\mathrm{Ef} \not\perp\!\!\!\perp \mathrm{Gr} \mid \{Sc\}$	$p = 0.00$	$\mathcal{S} = 0.50$
$\mathrm{Su} \not\perp\!\!\!\perp \mathrm{Gr} \mid \{Ef\}$	$p = 0.00$	$\mathcal{S} = 0.50$

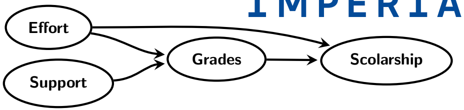
$\mathrm{Gr} \not\perp\!\!\!\perp \mathrm{Sc} \mid \{Ef\}$	$p = 0.00$	$\mathcal{S} = 0.50$
$\mathrm{Gr} \not\perp\!\!\!\perp \mathrm{Sc} \mid \{Su\}$	$p = 0.00$	$\mathcal{S} = 0.50$
$\mathrm{Su} \perp\!\!\!\perp \mathrm{Ef} \mid \{Gr\}$	$p = 0.53$	$\mathcal{S} = 0.38$
$\mathrm{Su} \not\perp\!\!\!\perp \mathrm{Sc} \mid \{Gr\}$	$p = 0.03$	$\mathcal{S} = 0.35$
$\mathrm{Su} \perp\!\!\!\perp \mathrm{Ef} \mid \{Gr\}$	$p = 0.33$	$\mathcal{S} = 0.32$
$\mathrm{Su} \perp\!\!\!\perp \mathrm{Ef} \mid \{Sc\}$	$p = 0.05$	$\mathcal{S} = 0.25$
~~$\mathrm{Su} \perp\!\!\!\perp \mathrm{Ef} \mid \{Gr, Sc\}$~~	~~$p = 0.39$~~	~~$\mathcal{S} = 0.00$~~
~~$\mathrm{Su} \not\perp\!\!\!\perp \mathrm{Sc} \mid \{Ef, Gr\}$~~	~~$p = 0.00$~~	~~$\mathcal{S} = 0.00$~~
~~$\mathrm{Ef} \not\perp\!\!\!\perp \mathrm{Gr} \mid \{Su, Sc\}$~~	~~$p = 0.00$~~	~~$\mathcal{S} = 0.00$~~
~~$\mathrm{Su} \not\perp\!\!\!\perp \mathrm{Sc} \mid \{Ef, Gr\}$~~	~~$p = 0.00$~~	~~$\mathcal{S} = 0.00$~~
~~$\mathrm{Su} \not\perp\!\!\!\perp \mathrm{Gr} \mid \{Ef, Sc\}$~~	~~$p = 0.03$~~	~~$\mathcal{S} = 0.00$~~

Student's Achievement Example



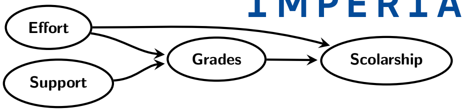
Ef $\not\perp\!\!\!\perp$ Sc	$p = 0.00$	$\mathcal{S} = 1.00$			
Ef $\not\perp\!\!\!\perp$ Gr	$p = 0.00$	$\mathcal{S} = 1.00$	Gr $\not\perp\!\!\!\perp$ Sc $\mid \{Ef\}$	$p = 0.00$	$\mathcal{S} = 0.50$
Su $\not\perp\!\!\!\perp$ Gr	$p = 0.00$	$\mathcal{S} = 1.00$	Gr $\not\perp\!\!\!\perp$ Sc $\mid \{Su\}$	$p = 0.00$	$\mathcal{S} = 0.50$
Gr $\not\perp\!\!\!\perp$ Sc	$p = 0.00$	$\mathcal{S} = 1.00$	Su $\perp\!\!\!\perp$ Ef $\mid \{Gr\}$	$p = 0.53$	$\mathcal{S} = 0.38$
Su $\perp\!\!\!\perp$ Ef	$p = 0.46$	$\mathcal{S} = 0.71$	Su $\not\perp\!\!\!\perp$ Sc $\mid \{Gr\}$	$p = 0.03$	$\mathcal{S} = 0.35$
Su $\not\perp\!\!\!\perp$ Sc	$p = 0.05$	$\mathcal{S} = 0.52$	Su $\perp\!\!\!\perp$ Ef $\mid \{Gr\}$	$p = 0.33$	$\mathcal{S} = 0.32$
Ef $\not\perp\!\!\!\perp$ Sc $\mid \{Su\}$	$p = 0.00$	$\mathcal{S} = 0.50$	~~Su $\perp\!\!\!\perp$ Ef $\mid \{Sc\}$~~	~~$p = 0.05$~~	~~$\mathcal{S} = 0.25$~~
Ef $\not\perp\!\!\!\perp$ Sc $\mid \{Gr\}$	$p = 0.00$	$\mathcal{S} = 0.50$	~~Su $\perp\!\!\!\perp$ Ef $\mid \{Gr, Sc\}$~~	~~$p = 0.39$~~	~~$\mathcal{S} = 0.00$~~
Ef $\not\perp\!\!\!\perp$ Gr $\mid \{Su\}$	$p = 0.00$	$\mathcal{S} = 0.50$	~~Su $\not\perp\!\!\!\perp$ Sc $\mid \{Ef, Gr\}$~~	~~$p = 0.00$~~	~~$\mathcal{S} = 0.00$~~
Su $\not\perp\!\!\!\perp$ Gr $\mid \{Sc\}$	$p = 0.00$	$\mathcal{S} = 0.50$	~~Ef $\not\perp\!\!\!\perp$ Gr $\mid \{Su, Sc\}$~~	~~$p = 0.00$~~	~~$\mathcal{S} = 0.00$~~
Ef $\not\perp\!\!\!\perp$ Gr $\mid \{Sc\}$	$p = 0.00$	$\mathcal{S} = 0.50$	~~Su $\not\perp\!\!\!\perp$ Sc $\mid \{Ef, Gr\}$~~	~~$p = 0.00$~~	~~$\mathcal{S} = 0.00$~~
Su $\not\perp\!\!\!\perp$ Gr $\mid \{Ef\}$	$p = 0.00$	$\mathcal{S} = 0.50$	~~Su $\not\perp\!\!\!\perp$ Gr $\mid \{Ef, Sc\}$~~	~~$p = 0.03$~~	~~$\mathcal{S} = 0.00$~~

Student's Achievement Example



$\mathrm{Ef} \not\perp\!\!\!\perp \mathrm{Sc}$	$p = 0.00$	$\mathcal{S} = 1.00$
$\mathrm{Ef} \not\perp\!\!\!\perp \mathrm{Gr}$	$p = 0.00$	$\mathcal{S} = 1.00$
$\mathrm{Su} \not\perp\!\!\!\perp \mathrm{Gr}$	$p = 0.00$	$\mathcal{S} = 1.00$
$\mathrm{Gr} \not\perp\!\!\!\perp \mathrm{Sc}$	$p = 0.00$	$\mathcal{S} = 1.00$
$\mathrm{Su} \perp\!\!\!\perp \mathrm{Ef}$	$p = 0.46$	$\mathcal{S} = 0.71$
$\mathrm{Su} \not\perp\!\!\!\perp \mathrm{Sc}$	$p = 0.05$	$\mathcal{S} = 0.52$
$\mathrm{Ef} \not\perp\!\!\!\perp \mathrm{Sc} \mid \{Su\}$	$p = 0.00$	$\mathcal{S} = 0.50$
$\mathrm{Ef} \not\perp\!\!\!\perp \mathrm{Sc} \mid \{Gr\}$	$p = 0.00$	$\mathcal{S} = 0.50$
$\mathrm{Ef} \not\perp\!\!\!\perp \mathrm{Gr} \mid \{Su\}$	$p = 0.00$	$\mathcal{S} = 0.50$
$\mathrm{Su} \not\perp\!\!\!\perp \mathrm{Gr} \mid \{Sc\}$	$p = 0.00$	$\mathcal{S} = 0.50$
$\mathrm{Ef} \not\perp\!\!\!\perp \mathrm{Gr} \mid \{Sc\}$	$p = 0.00$	$\mathcal{S} = 0.50$
$\mathrm{Su} \not\perp\!\!\!\perp \mathrm{Gr} \mid \{Ef\}$	$p = 0.00$	$\mathcal{S} = 0.50$

$\mathrm{Gr} \not\perp\!\!\!\perp \mathrm{Sc} \mid \{Ef\}$	$p = 0.00$	$\mathcal{S} = 0.50$
$\mathrm{Gr} \not\perp\!\!\!\perp \mathrm{Sc} \mid \{Su\}$	$p = 0.00$	$\mathcal{S} = 0.50$
$\mathrm{Su} \perp\!\!\!\perp \mathrm{Ef} \mid \{Gr\}$	$p = 0.53$	$\mathcal{S} = 0.38$
$\mathrm{Su} \not\perp\!\!\!\perp \mathrm{Sc} \mid \{Gr\}$	$p = 0.03$	$\mathcal{S} = 0.35$
~~$\mathrm{Su} \perp\!\!\!\perp \mathrm{Ef} \mid \{Gr\}$~~	~~$p = 0.33$~~	~~$\mathcal{S} = 0.32$~~
~~$\mathrm{Su} \perp\!\!\!\perp \mathrm{Ef} \mid \{Sc\}$~~	~~$p = 0.05$~~	~~$\mathcal{S} = 0.25$~~
~~$\mathrm{Su} \perp\!\!\!\perp \mathrm{Ef} \mid \{Gr, Sc\}$~~	~~$p = 0.39$~~	~~$\mathcal{S} = 0.00$~~
~~$\mathrm{Su} \not\perp\!\!\!\perp \mathrm{Sc} \mid \{Ef, Gr\}$~~	~~$p = 0.00$~~	~~$\mathcal{S} = 0.00$~~
~~$\mathrm{Ef} \not\perp\!\!\!\perp \mathrm{Gr} \mid \{Su, Sc\}$~~	~~$p = 0.00$~~	~~$\mathcal{S} = 0.00$~~
~~$\mathrm{Su} \not\perp\!\!\!\perp \mathrm{Sc} \mid \{Ef, Gr\}$~~	~~$p = 0.00$~~	~~$\mathcal{S} = 0.00$~~
~~$\mathrm{Su} \not\perp\!\!\!\perp \mathrm{Gr} \mid \{Ef, Sc\}$~~	~~$p = 0.03$~~	~~$\mathcal{S} = 0.00$~~



Student's Achievement Example

$Ef \not\perp\!\!\!\perp Sc$	$p = 0.00$	$\mathcal{S} = 1.00$			
$Ef \not\perp\!\!\!\perp Gr$	$p = 0.00$	$\mathcal{S} = 1.00$	$Gr \not\perp\!\!\!\perp Sc \mid \{Ef\}$	$p = 0.00$	$\mathcal{S} = 0.50$
$Su \not\perp\!\!\!\perp Gr$	$p = 0.00$	$\mathcal{S} = 1.00$	$Gr \not\perp\!\!\!\perp Sc \mid \{Su\}$	$p = 0.00$	$\mathcal{S} = 0.50$
$Gr \not\perp\!\!\!\perp Sc$	$p = 0.00$	$\mathcal{S} = 1.00$	$Su \perp\!\!\!\perp Ef \mid \{Gr\}$	$p = 0.53$	$\mathcal{S} = 0.38$
$Su \perp\!\!\!\perp Ef$	$p = 0.46$	$\mathcal{S} = 0.71$	~~$Su \not\perp\!\!\!\perp Sc \mid \{Gr\}$~~	~~$p = 0.03$~~	~~$\mathcal{S} = 0.35$~~
$Su \not\perp\!\!\!\perp Sc$	$p = 0.05$	$\mathcal{S} = 0.52$	~~$Su \perp\!\!\!\perp Ef \mid \{Gr\}$~~	~~$p = 0.33$~~	~~$\mathcal{S} = 0.32$~~
$Ef \not\perp\!\!\!\perp Sc \mid \{Su\}$	$p = 0.00$	$\mathcal{S} = 0.50$	~~$Su \perp\!\!\!\perp Ef \mid \{Sc\}$~~	~~$p = 0.05$~~	~~$\mathcal{S} = 0.25$~~
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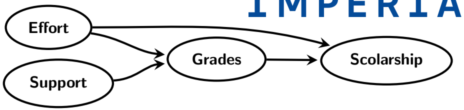
$Ef \not\perp\!\!\!\perp Gr \mid \{Su\}$	$p = 0.00$	$\mathcal{S} = 0.50$	~~$Su \not\perp\!\!\!\perp Sc \mid \{Ef, Gr\}$~~	~~$p = 0.00$~~	~~$\mathcal{S} = 0.00$~~
$Su \not\perp\!\!\!\perp Gr \mid \{Sc\}$	$p = 0.00$	$\mathcal{S} = 0.50$	~~$Ef \not\perp\!\!\!\perp Gr \mid \{Su, Sc\}$~~	~~$p = 0.00$~~	~~$\mathcal{S} = 0.00$~~
$Ef \not\perp\!\!\!\perp Gr \mid \{Sc\}$	$p = 0.00$	$\mathcal{S} = 0.50$	~~$Su \not\perp\!\!\!\perp Sc \mid \{Ef, Gr\}$~~	~~$p = 0.00$~~	~~$\mathcal{S} = 0.00$~~
$Su \not\perp\!\!\!\perp Gr \mid \{Ef\}$	$p = 0.00$	$\mathcal{S} = 0.50$	~~$Su \not\perp\!\!\!\perp Gr \mid \{Ef, Sc\}$~~	~~$p = 0.03$~~	~~$\mathcal{S} = 0.00$~~

Student's Achievement Example



$\mathrm{Ef} \not\perp\!\!\!\perp \mathrm{Sc}$	$p = 0.00$	$\mathcal{S} = 1.00$
$\mathrm{Ef} \not\perp\!\!\!\perp \mathrm{Gr}$	$p = 0.00$	$\mathcal{S} = 1.00$
$\mathrm{Su} \not\perp\!\!\!\perp \mathrm{Gr}$	$p = 0.00$	$\mathcal{S} = 1.00$
$\mathrm{Gr} \not\perp\!\!\!\perp \mathrm{Sc}$	$p = 0.00$	$\mathcal{S} = 1.00$
$\mathrm{Su} \perp\!\!\!\perp \mathrm{Ef}$	$p = 0.46$	$\mathcal{S} = 0.71$
$\mathrm{Su} \not\perp\!\!\!\perp \mathrm{Sc}$	$p = 0.05$	$\mathcal{S} = 0.52$
$\mathrm{Ef} \not\perp\!\!\!\perp \mathrm{Sc} \mid \{Su\}$	$p = 0.00$	$\mathcal{S} = 0.50$
$\mathrm{Ef} \not\perp\!\!\!\perp \mathrm{Sc} \mid \{Gr\}$	$p = 0.00$	$\mathcal{S} = 0.50$
$\mathrm{Ef} \not\perp\!\!\!\perp \mathrm{Gr} \mid \{Su\}$	$p = 0.00$	$\mathcal{S} = 0.50$
$\mathrm{Su} \not\perp\!\!\!\perp \mathrm{Gr} \mid \{Sc\}$	$p = 0.00$	$\mathcal{S} = 0.50$
$\mathrm{Ef} \not\perp\!\!\!\perp \mathrm{Gr} \mid \{Sc\}$	$p = 0.00$	$\mathcal{S} = 0.50$
$\mathrm{Su} \not\perp\!\!\!\perp \mathrm{Gr} \mid \{Ef\}$	$p = 0.00$	$\mathcal{S} = 0.50$

$\mathrm{Gr} \not\perp\!\!\!\perp \mathrm{Sc} \mid \{Ef\}$	$p = 0.00$	$\mathcal{S} = 0.50$
$\mathrm{Gr} \not\perp\!\!\!\perp \mathrm{Sc} \mid \{Su\}$	$p = 0.00$	$\mathcal{S} = 0.50$
~~$\mathrm{Su} \perp\!\!\!\perp \mathrm{Ef} \mid \{Gr\}$~~	~~$p = 0.53$~~	~~$\mathcal{S} = 0.38$~~
~~$\mathrm{Su} \not\perp\!\!\!\perp \mathrm{Sc} \mid \{Gr\}$~~	~~$p = 0.03$~~	~~$\mathcal{S} = 0.35$~~
~~$\mathrm{Su} \perp\!\!\!\perp \mathrm{Ef} \mid \{Gr\}$~~	~~$p = 0.33$~~	~~$\mathcal{S} = 0.32$~~
~~$\mathrm{Su} \perp\!\!\!\perp \mathrm{Ef} \mid \{Sc\}$~~	~~$p = 0.05$~~	~~$\mathcal{S} = 0.25$~~
~~$\mathrm{Su} \perp\!\!\!\perp \mathrm{Ef} \mid \{Gr, Sc\}$~~	~~$p = 0.39$~~	~~$\mathcal{S} = 0.00$~~
~~$\mathrm{Su} \not\perp\!\!\!\perp \mathrm{Sc} \mid \{Ef, Gr\}$~~	~~$p = 0.00$~~	~~$\mathcal{S} = 0.00$~~
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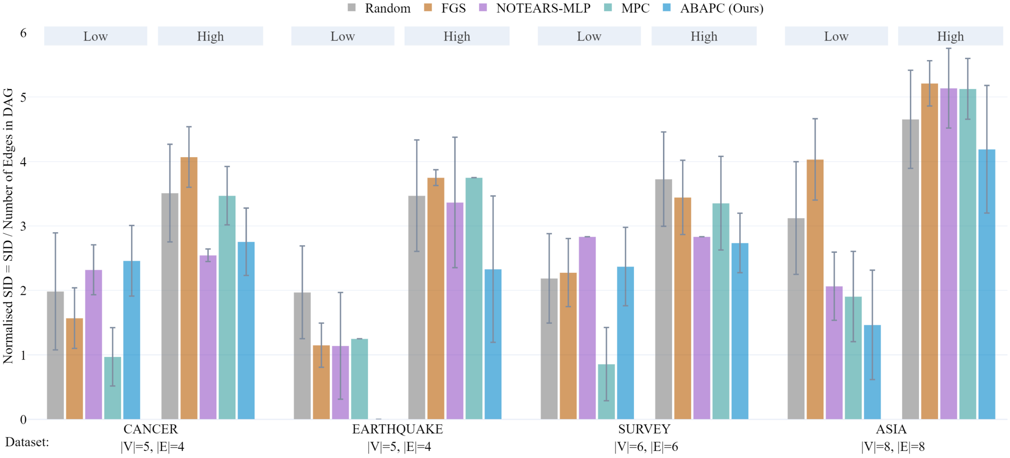
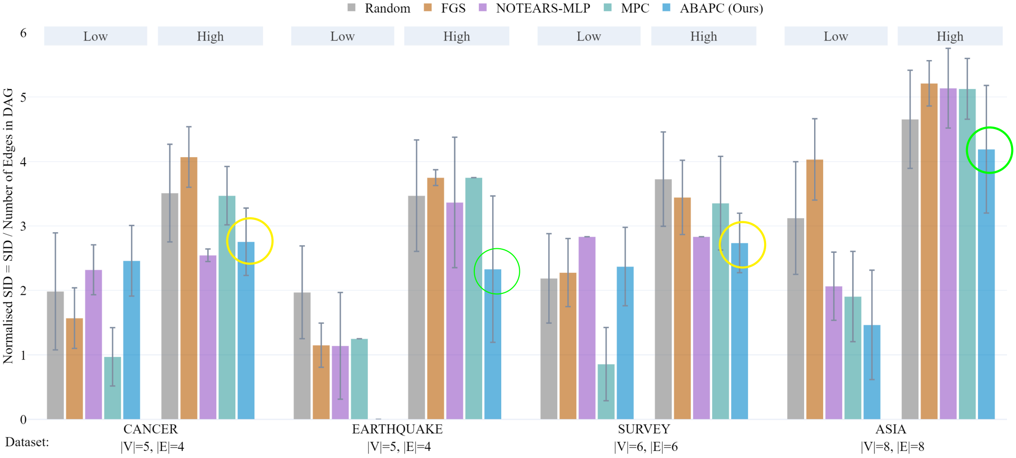
~~$\mathrm{Su} \not\perp\!\!\!\perp \mathrm{Sc} \mid \{Ef, Gr\}$~~	~~$p = 0.00$~~	~~$\mathcal{S} = 0.00$~~
~~$\mathrm{Su} \not\perp\!\!\!\perp \mathrm{Gr} \mid \{Ef, Sc\}$~~	~~$p = 0.03$~~	~~$\mathcal{S} = 0.00$~~

Empirical Evaluation

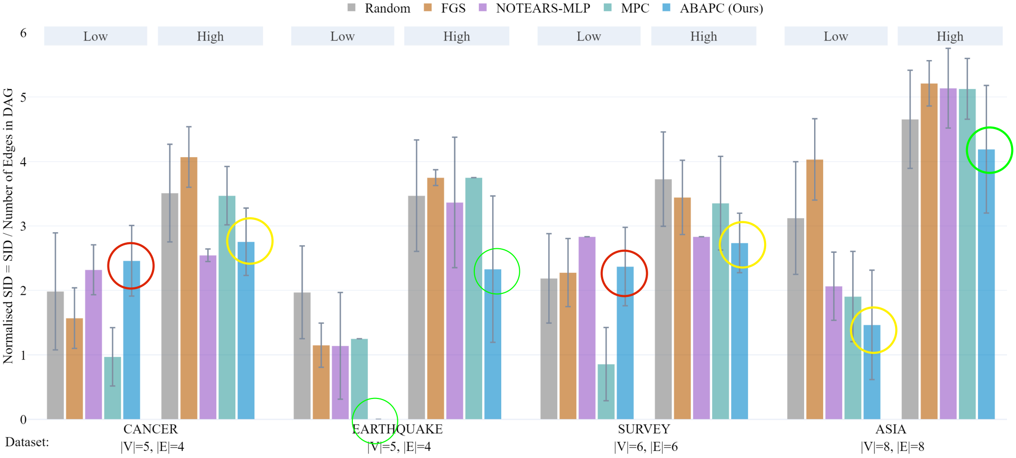
Structural Interventional Distance (Peters and Bühlmann 2015)



Structural Interventional Distance (Peters and Bühlmann 2015)



Structural Interventional Distance (Peters and Bühlmann 2015)



Conclusion & Future Work

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- We introduced Causal ABA - a framework to extract causal graphs from data with provable guarantees
  - We enforce correspondence between a set of independence tests and DAGs
  - Compare favourably to state-of-the-art on common benchmarks

Conclusions & Future Work

- We introduced Causal ABA - a framework to extract causal graphs from data with provable guarantees
  - We enforce correspondence between a set of independence tests and DAGs
  - Compare favourably to state-of-the-art on common benchmarks
- We are working on improving the scalability and applicability of the framework (e.g. latent features)
- Explainability, experts input, multiple algorithms, different semantics, repair, belief revision, Bayesian methods

Thanks for your attention!

Questions?

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Appendices

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Causal Effects Estimation

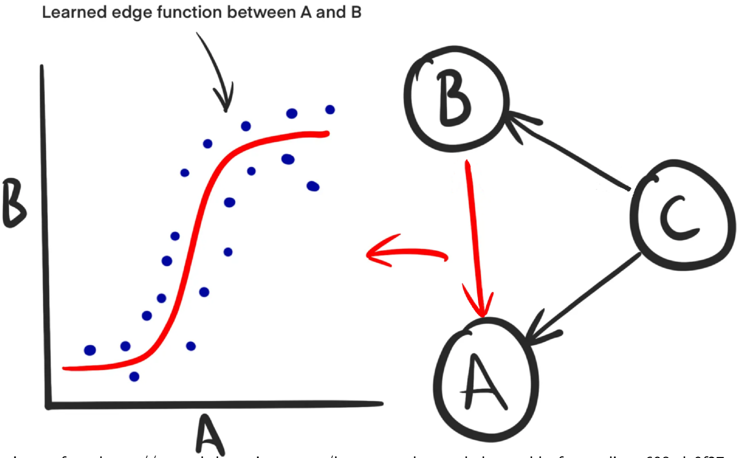

Image from https://towardsdatascience.com/how-to-understand-the-world-of-causality-c698cdc9f27c

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Causal Discovery

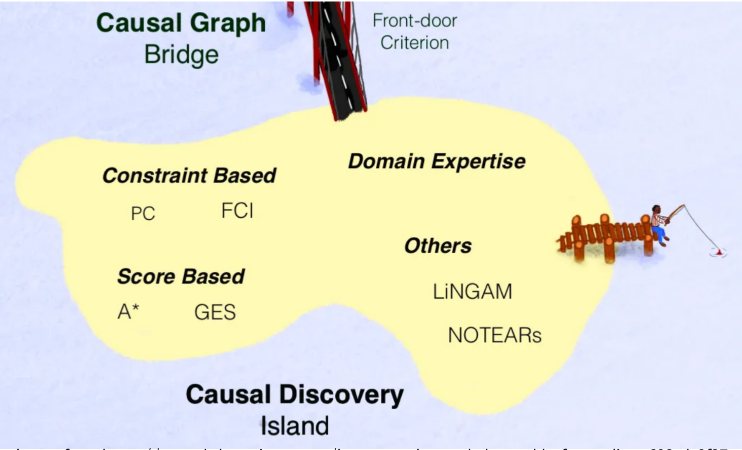
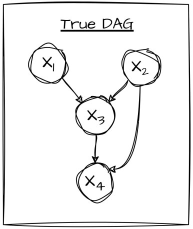


Image from https://towardsdatascience.com/how-to-understand-the-world-of-causality-c698cdc9f27c

PC-Algorithm (Spirtes, C. N. Glymour and Scheines 2000)

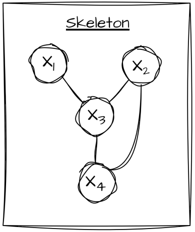


Input: Conditional Independence Information for all variables

1: Build skeleton $\mathcal{C}$ via adjacency search (save separating sets)
2: Orient v-structures in $\mathcal{C}$ (using separating sets from step 1)
3: Propagate *d-separation* via Meek rules

Return: CPDAG

PC-Algorithm (Spirtes, C. N. Glymour and Scheines 2000)



Input: Conditional Independence Information for all variables

1: **Build skeleton $\mathcal{C}$ via adjacency search (save separating sets)**
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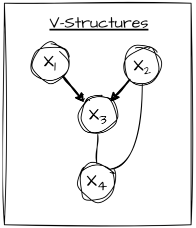

Input: Conditional Independence Information for all variables

1: Build skeleton $\mathcal{C}$ via adjacency search (save separating sets)
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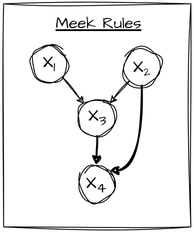
3: Propagate *d-separation* via Meek rules

Return: CPDAG

PC-Algorithm (Spirtes, C. N. Glymour and Scheines 2000)



Input: Conditional Independence Information for all variables

1: Build skeleton $\mathcal{C}$ via adjacency search (save separating sets)
2: Orient v-structures in $\mathcal{C}$ (using separating sets from step 1)
3: **Propagate d-separation via Meek rules**

Return: CPDAG

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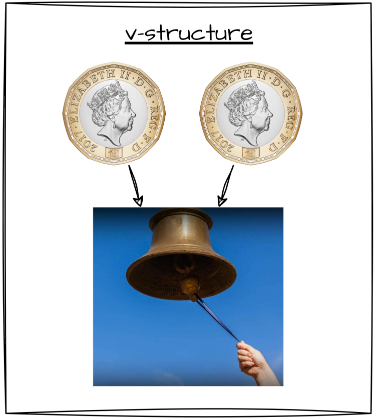
V-structures or Immoralities

- Two variables are marginally independent $Coin1 \perp\!\!\!\perp Coin2$
- But they are dependent when conditioned on a common descendant $Coin1 \not\!\perp\!\!\!\perp Coin2 \mid Bell$



PC-based Algorithms

1. Conservative-PC (Ramsey, Spirtes and Zhang 2006)
   → Orient v-structure only if the candidate collider renders dependence in **all** tests when in the conditioning set
2. Majority-PC (Colombo and Maathuis 2014)
   → Orient v-structure only if the candidate collider renders dependence in the **majority** of the tests when in the conditioning set
3. Max-PC (Ramsey 2016)
   → Orient v-structure only if the test with the **maximum p-value** does not contain the candidate collider
4. ML4C (Dai et al. 2023)
   → Build a machine learning model to **predict** v-structures
5. Shapley-PC (Russo and Toni 2023)
   → Orient v-structure only if the candidate collider has the **lowest Shapley Independence Value**

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PC-Algorithm (Propagation, Meek 1995)

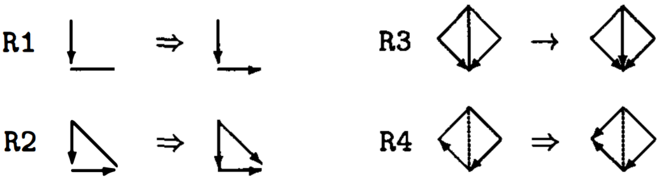


Figure 1: Orientation rules for patterns

Weighting Function

We re-base p-values around the significance level:

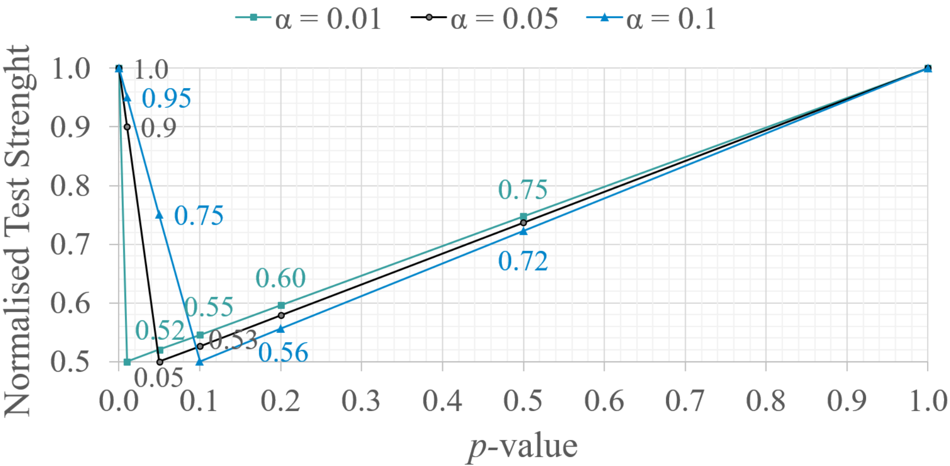
$$\gamma(p,\alpha) = \begin{cases} 2p\alpha - 1 & \text{iff } p < \alpha \\ \frac{2\alpha - p - 1}{2(\alpha - 1)} & \text{otherwise} \end{cases} \tag{1}$$

and penalise bigger conditioning set sizes:

$$\mathcal{S}(p,\alpha,s,d) = \frac{(1-s)}{(d-2)}\gamma(p,\alpha) \tag{2}$$

IMPERIAL

Weighting Function



Baselines

1. Majority-PC (Colombo and Maathuis 2014) is a modification of the PC-algorithm
2. Fast Greedy Equivalence Search (FGS) (Ramsey, M. Glymour et al. 2017) is a fast implementation of GES (Chickering 2002) where graphs are evaluated using the Bayesian Information Criterion
3. NOTEARS-MLP (Zheng et al. 2020) learns a non-linear SEM via continuous optimisation

Run Time Comparison